

Fermion Cooper Pairing with Unequal Masses: Standard Field Theory Approach

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The fermion Cooper pairing with unequal masses is investigated in a standard field theory approach. We derived the superfluid density and Meissner mass squared of the $U(1)$ gauge field in a general two species model and found that the often used proportional relation between the two quantities is broken down when the fermion masses are unequal. In weak coupling region, the superfluid density is always negative but the Meissner mass squared becomes mostly positive when the mass ratio between the pairing fermions is large enough. We established a proper momentum configuration of the LOFF pairing with unequal masses and showed that the LOFF state is energetically favored due to the negative superfluid density. The single plane wave LOFF state is physically equivalent to an anisotropic state with a spontaneously generated superflow. The extension to finite range interaction is briefly discussed.

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I. INTRODUCTION

The asymmetric cooper pairing between different species of fermions with mismatched Fermi surfaces, which was discussed many years ago, promoted new interest in both theoretic and experimental studies in recent years. The mismatched Fermi surfaces can be realized, for instance, in a superconductor with Zeeman splitting induced by an external field[1, 2, 3, 4], an atomic fermion gas composed of two species of atoms with different densities and/or masses[5, 6], an isospin asymmetric nuclear matter with proton-neutron pairing[7], and color superconducting quark matter with charge neutrality[8, 9, 10]. Among the mechanisms which can produce asymmetry between the pairing fermions, the mass difference is a very robust one. The cooper pairing between fermions with unequal masses was firstly investigated by V.Liu and F.Wilczek[11]. They considered a fermion gas composed of light and heavy fermions with attractive interaction. A homogeneous and isotropic pairing state which is similar to the Sarma state[1] was proposed to be the ground state of such systems. Such an exotic pairing state is now called breached pairing (BP) state or interior gap state. In the BP state there exists gapless fermion excitations, and the superfluid Fermi gas and the normal Fermi gas coexist in the momentum space.

It was found many years ago that the Sarma state suffers a thermodynamic instability[1]. It is now generally accepted that this Sarma instability can be cured in some physical conditions, such as a long-range interaction when charge neutrality is required[9, 10], a proper finite range interaction between the two species of fermions with large mass difference[12], and a superfluid Fermi gas with density imbalance in strong coupling region[13, 14, 15, 16]. While the Sarma instability can be cured, it was soon found that the superfluid density of the BP state is negative[17] and the free energy of the mixed phase is also lower than that of the BP state[18, 19]. Meanwhile, in the study of color superconductors possess paramagnetic response to external color

magnetic fields, i.e., the Meissner masses of some gluons are negative[20, 21, 22, 23, 24]. All these phenomena indicate that the homogeneous and isotropic BP state is unstable and some spatially inhomogeneous and isotropic states are energetically favored[25, 26, 27, 28, 29, 30, 31]. The superfluid density is a fundamental quantity in superconductivity. It is well known that the superfluid density is proportional to the Meissner mass squared and often measured via the London penetration depth in experiments. Due to this relation, one often regards the negative superfluid density observed in a BP state and the negative Meissner mass squared observed in a gapless color superconductor as the same instability.

However, almost all these studies focus on systems where the masses of the pairing fermions are equal. In the study of superfluid stability of interior gap states, S.T.Wu and S.Yip derived a formula of the superfluid density for non-relativistic asymmetric fermion superfluids with the concept of quasiparticles[17]. In the equal mass case, their formula is consistent with the result calculated from the linear response theory[32], the current-current correlation function[33], and the Meissner mass squared[34]. However, in unequal mass systems, the formula of superfluid density seems quite different from the Meissner mass squared[34]. Does the proportional relation between the superfluid density and the Meissner mass squared still hold in unequal mass systems? In this paper, we will derive the superfluid density and the Meissner mass squared in unequal mass systems in a standard field theory approach where the superfluid density and the Meissner mass squared are treated in the same way.

The paper is organized as follows. In Section II, we briefly review the formalism of the two species model. In Section III, we derive the formula of the superfluid density and compare it with S.T.Wu and S.Yip's phenomenological method. In Section IV, we derive the Meissner mass squared and show that it is not proportional to the superfluid density in unequal mass systems. The superfluid density and Meissner mass squared in the breached pairing states are calculated in Section V. The LOFF

pairing in unequal mass systems is discussed in Section VI. The extension to finite range pairing interaction is briefly discussed in Section VII. We summarize in Section VIII. We use the natural unit of $c = \hbar = k_B = 1$ through the paper.

II. TWO SPECIES MODEL

The physical system we are interested in in this paper is an idea system composed of two species of fermions with attractive interaction. The system is described by the Lagrangian density with imaginary time $\tau = it$,

$$\mathcal{L} = \sum_{i=a,b} \psi_i^* \left(-\partial_\tau + \frac{\nabla^2}{2m_i} + \mu_i \right) \psi_i + g \psi_a^* \psi_b^* \psi_b \psi_a, \quad (1)$$

where $\psi_i \equiv \psi_i(x)$ with $x = (\tau, \vec{x})$ are fermion fields for the two species a and b , the coupling constant g is positive to keep the interaction attractive, m_a and m_b are the masses for the two species, and μ_a and μ_b the chemical potentials.

The key quantity to describe a thermodynamic system is the partition function Z . It can be expressed as

$$Z = \int [d\psi_i][d\psi_i^*] e^{\int_0^\beta d\tau \int d^3\mathbf{x} \mathcal{L}[\psi_i, \psi_i^*]} \quad (2)$$

in the imaginary time formalism of finite temperature field theory, where β is the inverse of the temperature T , $\beta = 1/T$. For attractive interaction g , we can perform an exact Hubbard-Stratonovich transformation to introduce the auxiliary boson field $\phi(x)$ and its complex conjugate $\phi^*(x)$. With the Nambu-Gorkov fields $\Psi, \bar{\Psi}$ defined as

$$\Psi(x) = \begin{pmatrix} \psi_a \\ \psi_b^* \end{pmatrix}, \quad \bar{\Psi}(x) = (\psi_a^* \quad \psi_b), \quad (3)$$

we can express the partition function as

$$Z = \int [d\Psi][d\bar{\Psi}][d\phi][d\phi^*] e^{\int_0^\beta d\tau \int d^3\mathbf{x} (\bar{\Psi} \mathcal{K} \Psi - |\phi|^2/g)}, \quad (4)$$

where the kernel $\mathcal{K}[\phi, \phi^*]$ is defined as

$$\mathcal{K}[\phi, \phi^*] = \begin{pmatrix} -\partial_\tau + \frac{\nabla^2}{2m_a} + \mu_a & \phi \\ \phi^* & -\partial_\tau - \frac{\nabla^2}{2m_b} - \mu_b \end{pmatrix}. \quad (5)$$

In mean field approximation, we replace ϕ and ϕ^* by their ensemble averages Δ and Δ^* . In a homogenous and isotropic state, they are independent of coordinates. Then we can directly evaluate the Gaussian path integral and obtain the thermodynamic potential

$$\Omega = \frac{|\Delta|^2}{g} - T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \text{Tr} \ln \mathcal{G}^{-1}(i\omega_n, \mathbf{p}) \quad (6)$$

in terms of the inverse fermion propagator

$$\mathcal{G}^{-1}(i\omega_n, \mathbf{p}) = \begin{pmatrix} i\omega_n - \epsilon_{\mathbf{p}}^a & \Delta \\ \Delta^* & i\omega_n + \epsilon_{\mathbf{p}}^b \end{pmatrix} \quad (7)$$

with the free fermion dispersions $\epsilon_{\mathbf{p}}^i = \mathbf{p}^2/(2m_i) - \mu_i$. The explicit form of the fermion propagator which we need in the following sections can be explicitly expressed as

$$\mathcal{G}(i\omega_n, \mathbf{p}) = \begin{pmatrix} \mathcal{G}_{11}(i\omega_n, \mathbf{p}) & \mathcal{G}_{12}(i\omega_n, \mathbf{p}) \\ \mathcal{G}_{21}(i\omega_n, \mathbf{p}) & \mathcal{G}_{22}(i\omega_n, \mathbf{p}) \end{pmatrix} \quad (8)$$

with the matrix elements

$$\begin{aligned} \mathcal{G}_{11}(i\omega_n, \mathbf{p}) &= \frac{i\omega_n - \epsilon_A + \epsilon_S}{(i\omega_n - \epsilon_A)^2 - \epsilon_\Delta^2}, \\ \mathcal{G}_{22}(i\omega_n, \mathbf{p}) &= \frac{i\omega_n - \epsilon_A - \epsilon_S}{(i\omega_n - \epsilon_A)^2 - \epsilon_\Delta^2}, \\ \mathcal{G}_{12}(i\omega_n, \mathbf{p}) &= \frac{-\Delta}{(i\omega_n - \epsilon_A)^2 - \epsilon_\Delta^2}, \\ \mathcal{G}_{21}(i\omega_n, \mathbf{p}) &= \frac{-\Delta^*}{(i\omega_n - \epsilon_A)^2 - \epsilon_\Delta^2}, \end{aligned} \quad (9)$$

where the quantities ϵ_S, ϵ_A and ϵ_Δ are defined as

$$\epsilon_{S,A} = (\epsilon_{\mathbf{p}}^a \pm \epsilon_{\mathbf{p}}^b)/2, \quad \epsilon_\Delta = \sqrt{\epsilon_S^2 + |\Delta|^2}. \quad (10)$$

Since all quantities depend only on $|\Delta|$, we can set Δ to be real from now on. From the pole of the fermion propagator we can read the dispersions $\epsilon_{\mathbf{p}}^A$ and $\epsilon_{\mathbf{p}}^B$ of fermionic quasiparticles:

$$\epsilon_{\mathbf{p}}^A = \epsilon_\Delta + \epsilon_A, \quad \epsilon_{\mathbf{p}}^B = \epsilon_\Delta - \epsilon_A. \quad (11)$$

For $\epsilon_A = 0$, we recover the well know BCS type excitation. The asymmetric part ϵ_A is the key quantity to produce exotic pairing states.

The occupation numbers of the two species of fermions can be calculated via the diagonal elements of the fermion propagator,

$$\begin{aligned} n_a(\mathbf{p}) &= T \lim_{\eta \rightarrow 0} \sum_n \mathcal{G}_{11}(i\omega_n, \mathbf{p}) e^{i\omega_n \eta}, \\ n_b(\mathbf{p}) &= -T \lim_{\eta \rightarrow 0} \sum_n \mathcal{G}_{22}(i\omega_n, \mathbf{p}) e^{-i\omega_n \eta}. \end{aligned} \quad (12)$$

Completing the Matsubara frequency summation, we obtain

$$\begin{aligned} n_a(\mathbf{p}) &= u_p^2 f(\epsilon_{\mathbf{p}}^A) + v_p^2 f(-\epsilon_{\mathbf{p}}^B), \\ n_b(\mathbf{p}) &= u_p^2 f(\epsilon_{\mathbf{p}}^B) + v_p^2 f(-\epsilon_{\mathbf{p}}^A) \end{aligned} \quad (13)$$

with the coherent coefficients $u_p^2 = (1 + \epsilon_S/\epsilon_\Delta)/2$ and $v_p^2 = (1 - \epsilon_S/\epsilon_\Delta)/2$. The particle number densities n_a and n_b for the species a and b are obtained by integrating $n_a(\mathbf{p})$ and $n_b(\mathbf{p})$ over momentum.

III. SUPERFLUID DENSITY

In this section we try to derive the superfluid density in a standard field theory approach. When the superfluid moves with a uniform but small velocity \mathbf{v}_s ,

the condensates transform as $\Delta \rightarrow \Delta e^{2i\mathbf{q} \cdot \mathbf{x}}$ and $\Delta^* \rightarrow \Delta^* e^{-2i\mathbf{q} \cdot \mathbf{x}}$ with the total momentum of the cooper pair $2\mathbf{q} = (m_a + m_b)\mathbf{v}_s$, and the fermion fields transform as $\psi_a \rightarrow \psi_a e^{i\mathbf{q}_a \cdot \mathbf{x}}$ and $\psi_b \rightarrow \psi_b e^{i\mathbf{q}_b \cdot \mathbf{x}}$ with the momenta of the two species $\mathbf{q}_a = m_a \mathbf{v}_s$ and $\mathbf{q}_b = m_b \mathbf{v}_s$ which satisfy $\mathbf{q}_a + \mathbf{q}_b = 2\mathbf{q}$. The superfluid density tensor ρ_{ij} is defined as[37]

$$\Omega(\mathbf{v}_s) = \Omega(\mathbf{0}) + \mathbf{j}_s \cdot \mathbf{v}_s + \frac{1}{2} \rho_{ij} (\mathbf{v}_s)_i (\mathbf{v}_s)_j + \dots \quad (14)$$

For a homogeneous and isotropic superfluid, we have $\rho_{ij} = \delta_{ij} \rho_s / 3$, and the above formula can be reduced to

$$\Omega(\mathbf{v}_s) = \Omega(\mathbf{0}) + \mathbf{j}_s \cdot \mathbf{v}_s + \frac{1}{2} \rho_s \mathbf{v}_s^2 + \dots, \quad (15)$$

where ρ_s is the superfluid density. When ρ_s is negative, the homogeneous and isotropic state is unstable and a state with spontaneously generated superflow which breaks the rotational symmetry is energetically favored.

After the transformation of the condensates and the fermion fields, the thermodynamic potential is changed as

$$\Omega(\mathbf{v}_s) = \frac{\Delta^2}{g} - T \sum_n \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \text{Tr} \ln \mathcal{G}_s^{-1}(i\omega_n, \mathbf{p}) \quad (16)$$

in terms of the \mathbf{v}_s -dependent inverse propagator

$$\mathcal{G}_s^{-1}(i\omega_n, \mathbf{p}) = \begin{pmatrix} i\omega_n - \epsilon_{\mathbf{p}+\mathbf{q}_a}^a & \Delta \\ \Delta & i\omega_n + \epsilon_{\mathbf{p}-\mathbf{q}_b}^b \end{pmatrix}. \quad (17)$$

Using the relation

$$\mathcal{G}_s^{-1} = \mathcal{G}^{-1} - \mathbf{p} \cdot \mathbf{v}_s - \frac{1}{2} \Sigma_m \mathbf{v}_s^2 \quad (18)$$

with the matrix $\Sigma_m = \text{diag}(m_a, -m_b)$, we can do the derivative expansion

$$\begin{aligned} \text{Tr} \ln \mathcal{G}_s^{-1} - \text{Tr} \ln \mathcal{G}^{-1} &= \mathbf{p} \cdot \mathbf{v}_s \text{Tr}(\mathcal{G}) - \frac{\mathbf{v}_s^2}{2} \text{Tr}(\mathcal{G} \Sigma_m) \\ &- \frac{1}{2} (\mathbf{p} \cdot \mathbf{v}_s)^2 \text{Tr}(\mathcal{G} \mathcal{G}) + \dots \end{aligned} \quad (19)$$

With this expansion, we can expand $\Omega(\mathbf{v}_s)$ in powers of \mathbf{v}_s . The superfluid density can be read from the quadratic term in \mathbf{v}_s . After some direct algebras, we obtain

$$\rho_s = m_a n_a + m_b n_b + \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{p^2}{3} (\mathcal{T}_{11} + \mathcal{T}_{22} + 2\mathcal{T}_{12}) \quad (20)$$

where $\mathcal{T}_{11}, \mathcal{T}_{22}, \mathcal{T}_{12}$ are the fermion Matsubara frequency

summations defined as

$$\begin{aligned} \mathcal{T}_{11} &= T \sum_n \mathcal{G}_{11} \mathcal{G}_{11} \\ &= u_p^2 v_p^2 \frac{f(\epsilon_{\mathbf{p}}^A) + f(\epsilon_{\mathbf{p}}^B) - 1}{\epsilon_{\Delta}} + u_p^4 f'(\epsilon_{\mathbf{p}}^A) + v_p^4 f'(\epsilon_{\mathbf{p}}^B), \\ \mathcal{T}_{22} &= T \sum_n \mathcal{G}_{22} \mathcal{G}_{22} \\ &= u_p^2 v_p^2 \frac{f(\epsilon_{\mathbf{p}}^A) + f(\epsilon_{\mathbf{p}}^B) - 1}{\epsilon_{\Delta}} + v_p^4 f'(\epsilon_{\mathbf{p}}^A) + u_p^4 f'(\epsilon_{\mathbf{p}}^B), \\ \mathcal{T}_{12} &= T \sum_n \mathcal{G}_{12} \mathcal{G}_{21} \\ &= u_p^2 v_p^2 \left[\frac{1 - f(\epsilon_{\mathbf{p}}^A) - f(\epsilon_{\mathbf{p}}^B)}{\epsilon_{\Delta}} + f'(\epsilon_{\mathbf{p}}^A) + f'(\epsilon_{\mathbf{p}}^B) \right] \end{aligned} \quad (21)$$

with $f(x)$ being the Fermi distribution function and $f'(x) = df(x)/dx$. Using these results we get

$$\rho_s = m_a n_a + m_b n_b + \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{p^2}{3} [f'(\epsilon_{\mathbf{p}}^A) + f'(\epsilon_{\mathbf{p}}^B)] \quad (22)$$

One can easily check that this formula is invariant under the exchange $a \leftrightarrow b$. When $\Delta = 0$, i.e., in the normal state, we have

$$\begin{aligned} \rho_s &= \int_0^\infty dp \frac{p^2}{2\pi^2} [m_a f(\epsilon_{\mathbf{p}}^a) + m_b f(\epsilon_{\mathbf{p}}^b)] \\ &+ \int_0^\infty dp \frac{p^4}{6\pi^2} [f'(\epsilon_{\mathbf{p}}^a) + f'(\epsilon_{\mathbf{p}}^b)]. \end{aligned} \quad (23)$$

From the identity

$$\int_0^\infty dp \frac{p^4}{6\pi^2} f'(\epsilon_{\mathbf{p}}^i) = -m_i \int_0^\infty dp \frac{p^2}{2\pi^2} f(\epsilon_{\mathbf{p}}^i), \quad (24)$$

ρ_s vanishes automatically in the normal state.

The result we obtained here is in agreement with the formula derived by S.T.Wu and S.Yip with a phenomenological method[17]. With their method, in presence of a small superfluid velocity \mathbf{v}_s , the quasiparticle energies are shifted by $\mathbf{p} \cdot \mathbf{v}_s$ and the occupation numbers become

$$\begin{aligned} \tilde{n}_a(\mathbf{p}) &= u_p^2 f(\epsilon_{\mathbf{p}}^A + \mathbf{p} \cdot \mathbf{v}_s) + v_p^2 f(-\epsilon_{\mathbf{p}}^B + \mathbf{p} \cdot \mathbf{v}_s), \\ \tilde{n}_b(\mathbf{p}) &= u_p^2 f(\epsilon_{\mathbf{p}}^B + \mathbf{p} \cdot \mathbf{v}_s) + v_p^2 f(-\epsilon_{\mathbf{p}}^A + \mathbf{p} \cdot \mathbf{v}_s). \end{aligned} \quad (25)$$

The number current can be decomposed into a diamagnetic and a paramagnetic parts:

$$\begin{aligned} \mathbf{J}_i^d &= \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \tilde{n}_i(\mathbf{p}) \mathbf{v}_s \equiv \rho_i^d \mathbf{v}_s, \\ \mathbf{J}_i^p &= \frac{1}{m_i} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \mathbf{p} \tilde{n}_i(\mathbf{p}) \equiv \rho_i^p \mathbf{v}_s. \end{aligned} \quad (26)$$

To leading order in \mathbf{v}_s , we have $\rho_i^d = n_i$. Using the fact $\mathbf{J}_i^p = 0$ for $\mathbf{v}_s = 0$, i.e.,

$$\mathbf{J}_i^p = \frac{1}{m_i} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \mathbf{p} (\tilde{n}_i(\mathbf{p}) - n_i(\mathbf{p})), \quad (27)$$

we obtain

$$\begin{aligned}\rho_a^p &= \frac{1}{m_a} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^2}{3} [u_p^2 f'(\epsilon_{\mathbf{p}}^A) + v_p^2 f'(\epsilon_{\mathbf{p}}^B)], \\ \rho_b^p &= \frac{1}{m_b} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^2}{3} [u_p^2 f'(\epsilon_{\mathbf{p}}^B) + v_p^2 f'(\epsilon_{\mathbf{p}}^A)].\end{aligned}\quad (28)$$

The total superfluid density is defined as $\rho_s = m_a \rho_a + m_b \rho_b$ with $\rho_i = \rho_i^d + \rho_i^p$. Using the fact $u_p^2 + v_p^2 = 1$, it is exactly the formula we obtained above. From our derivation, we can also decompose the superfluid density into two parts

$$\begin{aligned}\rho_s^a &= m_a n_a + \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^2}{3} (\mathcal{T}_{11} + \mathcal{T}_{12}), \\ \rho_s^b &= m_b n_b + \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^2}{3} (\mathcal{T}_{22} + \mathcal{T}_{12}),\end{aligned}\quad (29)$$

where $\rho_s^a \equiv m_a \rho_a$ and $\rho_s^b \equiv m_b \rho_b$ can be defined as the superfluid densities for the two species of fermions.

One may ask why these two derivations give the same result. In presence of a small superflow \mathbf{v}_s , the quasiparticle dispersions can be read from $\det \mathcal{G}_s^{-1} = 0$. After a simple algebra, we get the modified dispersions for the quasiparticles

$$\begin{aligned}\tilde{\epsilon}_{\mathbf{p}}^A &= \sqrt{\tilde{\epsilon}_S^2 + \Delta^2} + \tilde{\epsilon}_A + \mathbf{p} \cdot \mathbf{v}_s, \\ \tilde{\epsilon}_{\mathbf{p}}^B &= \sqrt{\tilde{\epsilon}_S^2 + \Delta^2} - \tilde{\epsilon}_A - \mathbf{p} \cdot \mathbf{v}_s\end{aligned}\quad (30)$$

with

$$\begin{aligned}\tilde{\epsilon}_S &= \epsilon_S + \frac{1}{4}(m_a + m_b)\mathbf{v}_s^2, \\ \tilde{\epsilon}_A &= \epsilon_A + \frac{1}{4}(m_a - m_b)\mathbf{v}_s^2.\end{aligned}\quad (31)$$

To leading order in \mathbf{v}_s , the quasiparticle energies are really shifted by $\mathbf{p} \cdot \mathbf{v}_s$. However, one should note that the derivation with the concept of quasiparticle is inconsistent. The particle occupation numbers in presence of a superflow should be

$$\begin{aligned}\tilde{n}_a(\mathbf{p}) &= u_p^2 f(\epsilon_{\mathbf{p}}^A + \mathbf{p} \cdot \mathbf{v}_s) + v_p^2 f(-\epsilon_{\mathbf{p}}^B + \mathbf{p} \cdot \mathbf{v}_s), \\ \tilde{n}_b(\mathbf{p}) &= u_p^2 f(\epsilon_{\mathbf{p}}^B - \mathbf{p} \cdot \mathbf{v}_s) + v_p^2 f(-\epsilon_{\mathbf{p}}^A - \mathbf{p} \cdot \mathbf{v}_s)\end{aligned}\quad (32)$$

to leading order in \mathbf{v}_s . Using this correct occupation numbers, one can not obtain the correct result since ρ_b^p will change a sign. We guess that for such an asymmetric system, one can not self-consistently derive the superfluid density with the concept of quasiparticles. Only for symmetric systems like the standard BCS, the method works as discussed in text books.

The formula of the superfluid density we derived here is in principle only suitable for grand canonical ensembles where the chemical potentials μ_a and μ_b are fixed[38]. For systems where the particle numbers n_a and n_b are fixed or the total number $n = n_a + n_b$ is fixed, we should use the free energy $\mathcal{F} = \Omega + \mu_a n_a + \mu_b n_b$ or $\mathcal{F} = \Omega + (\mu_a + \mu_b)n/2$

instead of the thermodynamic potential Ω to calculate the superfluid density. However, we can show that such a correction is beyond order $O(\mathbf{v}_s^2)$ for an homogeneous and isotropic state[39], and hence we can safely apply the above formula to the systems with fixed particle numbers. For instance, if n_a and n_b is fixed, we have

$$\rho_s = \left. \frac{\partial^2 \mathcal{F}}{\partial \mathbf{v}_s^2} \right|_{\mathbf{v}_s=0} = \left. \frac{\partial^2 \Omega}{\partial \mathbf{v}_s^2} \right|_{\mathbf{v}_s=0} + \left. \frac{\partial n_i}{\partial \mathbf{v}_s} \frac{\partial \mu_i}{\partial \mathbf{v}_s} \right|_{\mathbf{v}_s=0}. \quad (33)$$

For a homogeneous and isotropic state, the term $\partial n_i / \partial \mathbf{v}_s|_{\mathbf{v}_s=0}$ vanishes automatically.

In our derivation, we did not use the assumption of weak coupling, and the formula can be applied to study the superfluid stability in the BCS-BEC crossover in a light-heavy fermion gas such as a mixture of ^6Li and ^{40}K . In recent studies on BCS-BEC crossover in equal mass systems, it was found that the BP state is stable in the BEC region, i.e., it is free from the Sarma instability and negative superfluid density[13]. We expect that such a stable BP state can also be realized in a light-heavy fermion gas in strong coupling.

IV. MEISSNER MASS

The two species model is invariant under the following phase transformation

$$\psi_i(x) \rightarrow e^{i\varphi_i} \psi_i(x), \quad \phi(x) \rightarrow e^{i(\varphi_a + \varphi_b)} \phi(x) \quad (34)$$

with arbitrary and constant phases φ_a and φ_b , which means that the symmetry group of the model is $U(1)_{\varphi_a} \otimes U(1)_{\varphi_b}$. The order parameter is invariant only for $\varphi_a = -\varphi_b$. In presence of a nonzero expectation value of ϕ , the symmetry group is spontaneously broken down to a $U(1)$ subgroup

$$U(1)_{\varphi_a} \otimes U(1)_{\varphi_b} \rightarrow U(1)_{\varphi_a - \varphi_b}. \quad (35)$$

The unbroken $U(1)$ subgroup corresponds to the phase difference $\Delta\varphi = \varphi_a - \varphi_b$, and a Goldstone mode corresponding to the total phase $\varphi = \varphi_a + \varphi_b$ will appear. Let's add a $U(1)$ gauge field A_μ in the Lagrangian,

$$\mathcal{L} = \sum_{i=a,b} \psi_i^* \left(-D_{\tau i} + \frac{\mathbf{D}_i^2}{2m_i} + \mu_i \right) \psi_i + g \psi_a^* \psi_b^* \psi_b \psi_a + \mathcal{L}_A \quad (36)$$

with $D_{\mu i} = \partial_\mu - ieQ_i A_\mu$, where \mathcal{L}_A is the gauge field sector, and eQ_a, eQ_b are the gauge couplings for the two species of fermions. In presence of a gauge field, the Goldstone mode disappears and the gauge field will obtain a mass m_A via Higgs mechanism. This is called Meissner effect in superconductivity, and the mass m_A the gauge field obtains is called Meissner mass.

The standard way to calculate the Meissner mass is to evaluate the polarization tensor $\Pi_{\mu\nu}(K)$ of the gauge field. For the interaction (36), the spatial components of the polarization tensor read

$$\Pi_{ij}(k) = \Pi_{ij}^d(k) + \Pi_{ij}^p(k) \quad (37)$$

with the diamagnetic part

$$\Pi_{ij}^d(k) = \delta_{ij} e^2 \frac{T}{V} \sum_p \text{Tr}[\mathcal{G}(p) \Sigma_d] \quad (38)$$

and the paramagnetic part

$$\Pi_{ij}^p(k) = e^2 \frac{T}{V} \sum_p p_i p_j \text{Tr}[\mathcal{G}(p_+) \Sigma_p \mathcal{G}(p_-) \Sigma_p] \quad (39)$$

with $p_{\pm} = p \pm k/2$, where the matrices Σ_d and Σ_p are defined as

$$\Sigma_d = \begin{pmatrix} \frac{Q_a^2}{m_a} & 0 \\ 0 & -\frac{Q_b^2}{m_b} \end{pmatrix}, \quad \Sigma_p = \begin{pmatrix} \frac{Q_a}{m_a} & 0 \\ 0 & \frac{Q_b}{m_b} \end{pmatrix}. \quad (40)$$

Note that the paramagnetic part is just the current-current correlation function which gives both diamagnetic part and paramagnetic part in relativistic systems[20, 21] but only paramagnetic part in non-relativistic systems. The Meissner mass m_A can be evaluated by

$$m_A^2 = \frac{1}{2} \lim_{\mathbf{k} \rightarrow 0} (\delta_{ij} - \hat{k}_i \hat{k}_j) \Pi_{ij}(0, \mathbf{k}). \quad (41)$$

If m_A^2 is negative, the homogeneous and isotropic state suffers the magnetic instability[20, 21] and a state with gauge field condensation $\langle \mathbf{A} \rangle \neq 0$ which breaks the rotational symmetry is energetically favored.

For a clear comparison of the superfluid density and the Meissner mass squared in unequal mass systems, we employ another approach[34]. The Meissner mass squared can be calculated via the response of the effective potential to an external transverse vector potential. In presence of a small external vector potential $\mathbf{A}(0, \mathbf{q} \rightarrow 0)$ in the static and long wave limit, the effective potential of the system can be expanded in powers of \mathbf{A} ,

$$\Omega(\mathbf{A}) = \Omega(0) + \mathbf{J}_A \cdot \mathbf{A} + \frac{1}{2} M_{ij}^2 \mathbf{A} \cdot \mathbf{A} + \dots \quad (42)$$

with the coefficients

$$M_{ij}^2 = \frac{\partial^2 \Omega(\mathbf{A})}{\partial A_i \partial A_j} \Big|_{\mathbf{A}=0}. \quad (43)$$

The coefficients M_{ij}^2 are just the components of the Meissner mass squared tensor. In a homogenous and isotropic superconductor, we have $M_{ij}^2 = 0$ for $i \neq j$ and $M_{11}^2 = M_{22}^2 = M_{33}^2$, and the Meissner mass squared m_A^2 is defined as

$$m_A^2 = \frac{1}{3} \sum_{i=1}^3 M_{ii}^2. \quad (44)$$

The thermodynamic potential in presence of the static and long wave vector potential \mathbf{A} can be expressed as

$$\Omega(\mathbf{A}) = \frac{\Delta^2}{g} - T \sum_n \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \text{Tr} \ln \mathcal{G}_A^{-1}(i\omega_n, \mathbf{p}) \quad (45)$$

with the \mathbf{A} -dependent inverse propagator

$$\mathcal{G}_A^{-1}(i\omega_n, \mathbf{p}) = \begin{pmatrix} i\omega_n - \epsilon_{\mathbf{p}+eQ_a\mathbf{A}}^a & \Delta \\ \Delta & i\omega_n + \epsilon_{\mathbf{p}-eQ_b\mathbf{A}}^b \end{pmatrix}. \quad (46)$$

Using the same trick in Section III, we have the relation

$$\mathcal{G}_A^{-1} = \mathcal{G}^{-1} - e \Sigma_p \mathbf{p} \cdot \mathbf{A} - \frac{e^2}{2} \Sigma_d \mathbf{A}^2 \quad (47)$$

and the derivative expansion

$$\begin{aligned} \text{Tr} \ln \mathcal{G}_A^{-1} - \text{Tr} \ln \mathcal{G}^{-1} &= e \mathbf{p} \cdot \mathbf{A} \text{Tr}(\mathcal{G} \Sigma_p) \\ &- \frac{e^2}{2} \mathbf{A}^2 \text{Tr}(\mathcal{G} \Sigma_d) - \frac{e^2}{2} (\mathbf{p} \cdot \mathbf{A})^2 \text{Tr}(\mathcal{G} \Sigma_p \mathcal{G} \Sigma_p) + \dots \end{aligned} \quad (48)$$

The Meissner mass squared can be read from the quadratic terms in \mathbf{A} . After some algebras we obtain

$$\begin{aligned} m_A^2 &= e^2 \left(\frac{n_a}{m_a} Q_a^2 + \frac{n_b}{m_b} Q_b^2 \right) \\ &+ e^2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{p^2}{3} \left(\frac{Q_a^2}{m_a^2} \mathcal{T}_{11} + \frac{Q_b^2}{m_b^2} \mathcal{T}_{22} + \frac{2Q_a Q_b}{m_a m_b} \mathcal{T}_{12} \right). \end{aligned} \quad (49)$$

The second term is just the long-wave and static limit of the current-current correlation function. With the result of frequency summations in Section III, we obtain an explicit expression,

$$\begin{aligned} m_A^2 &= e^2 \left(\frac{n_a}{m_a} Q_a^2 + \frac{n_b}{m_b} Q_b^2 \right) + e^2 \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{p^2}{3} \times \\ &\left[\left(\frac{Q_a}{m_a} - \frac{Q_b}{m_b} \right)^2 u_p^2 v_p^2 \frac{f(\epsilon_{\mathbf{p}}^A) + f(\epsilon_{\mathbf{p}}^B) - 1}{\epsilon_{\Delta}} + \left(\frac{Q_a}{m_a} u_p^2 + \frac{Q_b}{m_b} v_p^2 \right)^2 f'(\epsilon_{\mathbf{p}}^A) + \left(\frac{Q_a}{m_a} v_p^2 + \frac{Q_b}{m_b} u_p^2 \right)^2 f'(\epsilon_{\mathbf{p}}^B) \right]. \end{aligned} \quad (50)$$

The formula is invariant under the exchange $a \leftrightarrow b$. For $\Delta = 0$, it is reduced to

$$m_A^2 = e^2 \int_0^\infty dp \frac{p^2}{2\pi^2} \left[\frac{f(\epsilon_p^a)}{m_a} Q_a^2 + \frac{f(\epsilon_p^b)}{m_b} Q_b^2 \right] + e^2 \int_0^\infty dp \frac{p^4}{6\pi^2} \left[\frac{f'(\epsilon_p^a)}{m_a^2} Q_a^2 + \frac{f'(\epsilon_p^b)}{m_b^2} Q_b^2 \right] \quad (51)$$

and vanishes in the normal state, as we can expect.

The formulae of the superfluid density and the Meissner mass squared we derived seem quite different. In the symmetric case with $m_a = m_b \equiv m$ and $Q_a = Q_b = 1$, we recover the well know result [32, 33]

$$\begin{aligned}\rho_s &= mn + \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^2}{3} [f'(\epsilon_{\mathbf{p}}^A) + f'(\epsilon_{\mathbf{p}}^B)], \\ m_A^2 &= \frac{ne^2}{m} + \frac{e^2}{m^2} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^2}{3} [f'(\epsilon_{\mathbf{p}}^A) + f'(\epsilon_{\mathbf{p}}^B)],\end{aligned}\quad (52)$$

and the proportional relation

$$\frac{\rho_s}{m_A^2} = \frac{m^2}{e^2} \quad (53)$$

at any temperature $T < T_c$. In fact, one can easily check that for systems with $Q_a/m_a = Q_b/m_b$, the proportional relation still holds. However, for general asymmetric systems, this relation is broken down.

V. THE BREACHED PAIRING STATE

We calculate the superfluid density and the Meissner mass squared for the breached pairing state at zero temperature in this section. Explicitly, the dispersions of the fermionic quasiparticles can be expressed as

$$\epsilon_{\mathbf{p}}^{A,B} = \sqrt{\left(\frac{p^2}{2m} - \mu\right)^2 + \Delta^2} \pm \left(\frac{p^2}{2m'} + \delta\mu\right) \quad (54)$$

with the reduced masses $m = 2m_a m_b / (m_a + m_b)$ and $m' = 2m_a m_b / (m_b - m_a)$ and chemical potentials $\mu = (\mu_a + \mu_b)/2$ and $\delta\mu = (\mu_b - \mu_a)/2$. One can easily check that for

$$\Delta < \Delta_c = \frac{|p_b^2 - p_a^2|}{4\sqrt{m_a m_b}}, \quad (55)$$

with $p_i = \sqrt{2m_i \mu_i}$, one branch of the fermionic quasiparticles can cross the momentum axis and hence becomes gapless. This is the so called breached pairing state or interior gap state[11]. The gapless nodes determined by $\epsilon_{\mathbf{p}}^A = 0$ or $\epsilon_{\mathbf{p}}^B = 0$ are located at $p = p_{1,2}$ with

$$p_{1,2}^2 = \frac{p_a^2 + p_b^2}{2} \mp \frac{1}{2} \sqrt{(p_a^2 - p_b^2)^2 - 16m_a m_b \Delta^2}. \quad (56)$$

The gap equation which determines Δ at zero temperature reads

$$-\frac{m}{4\pi a_s} = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\frac{\Theta(\epsilon_{\mathbf{p}}^A) - \Theta(-\epsilon_{\mathbf{p}}^B)}{2\epsilon_{\Delta}} - \frac{1}{2\xi_p} \right] = 0. \quad (57)$$

Here $\Theta(x)$ is the step function, and the s-wave scattering length a_s is related to the bare coupling g via

$$\frac{m}{4\pi a_s} = -\frac{1}{g} + \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2\xi_p} \quad (58)$$

with $\xi_p = p^2/2m$. One can employ another regularization scheme where the pairing interaction is restricted in a narrow region around the common Fermi surface $p_F = \sqrt{2m\mu}$ [11] which is suitable only for weak coupling case. We have checked that the results from these two regularization schemes are the same in weak coupling.

In the BCS phase, all fermionic excitations are gapped, the gap equation can be reduced to

$$-\frac{m}{4\pi a_s} = \int_0^\infty dp \frac{p^2}{2\pi^2} \left(\frac{1}{2\epsilon_{\Delta}} - \frac{1}{2\xi_p} \right), \quad (59)$$

and the solution in weak coupling is

$$\Delta_0 \simeq 8e^{-2}\mu e^{-\frac{\pi}{2p_F|a_s|}}. \quad (60)$$

Since all fermionic quasiparticles are gapped, we have $n_a = n_b = n/2$, and the superfluid density reads

$$\rho_s = m_a n_a + m_b n_b = \frac{1}{2}(m_a + m_b)n, \quad (61)$$

which means that all fermions participate in the superfluid. In weak coupling, we have approximately $n \simeq \frac{p_F^3}{3\pi^2}$ and

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^2}{12} \frac{\Delta^2}{\epsilon_{\Delta}^3} \simeq \frac{mp_F^3}{12\pi^2} \quad (62)$$

which leads to

$$m_A^2 = \frac{ne^2(Q_a + Q_b)^2}{2(m_a + m_b)}. \quad (63)$$

Note that m_A^2 depends only on $Q_a + Q_b$, and this conclusion is valid only in weak coupling we considered by hand.

In the breached pairing state with gapless fermionic excitations, the gap equation can be expressed as

$$-\frac{m}{4\pi a_s} = \int_0^\infty dp \frac{p^2}{2\pi^2} \left(\frac{1}{2\epsilon_{\Delta}} - \frac{1}{2\xi_p} \right) - \int_{p_1}^{p_2} dp \frac{p^2}{2\pi^2} \frac{1}{2\epsilon_{\Delta}}. \quad (64)$$

Using the equation for the BCS gap Δ_0 , the solution of the gap equation can be well described by[19]

$$\Delta = \sqrt{\Delta_0(2\Delta_c - \Delta_0)}. \quad (65)$$

The gap varies in the region $0 < \Delta < \Delta_0$, and the difference between the Fermi momenta satisfies

$$2\sqrt{m_a m_b} \Delta_0 < |p_a^2 - p_b^2| < 4\sqrt{m_a m_b} \Delta_0. \quad (66)$$

If we define the density asymmetry $\alpha = |n_a - n_b|/(n_a + n_b)$, then $\Delta = \Delta_0$ corresponds to $\alpha = 0$ and $\Delta = 0$ corresponds to the maximal asymmetry α_c . When α varies from 0 to α_c , the gap Δ decreases from Δ_0 to 0.

We firstly discuss the case $m_a = m_b$ where the superfluid density is proportional to the Meissner mass squared. In this case we have $\Delta_c = \delta\mu$ and $p^2/2m' = 0$.

Let's set $\delta\mu > 0$ without loss of generality. At zero temperature the superfluid density reads

$$\rho_s = mn - \frac{1}{6\pi^2} \int_0^\infty dp p^4 \delta(\epsilon_\Delta - \delta\mu). \quad (67)$$

In the weak coupling region, $\epsilon_\Delta - \delta\mu = 0$ has two possible roots p_1, p_2 and ρ_s can be evaluated as

$$\rho_s = mn \left[1 - \eta \frac{\delta\mu \Theta(\delta\mu - \Delta)}{\sqrt{\delta\mu^2 - \Delta^2}} \right] \quad (68)$$

with the coefficient $\eta = (p_1^3 + p_2^3) / (6\pi^2 n)$. Since the coefficient η is approximately equal to 1, ρ_s can be well approximated by

$$\rho_s \simeq mn \left[1 - \frac{\delta\mu \Theta(\delta\mu - \Delta)}{\sqrt{\delta\mu^2 - \Delta^2}} \right]. \quad (69)$$

It is now clear that in the BP state with $\Delta < \delta\mu$, ρ_s becomes negative and the BP state is unstable. We should emphasize that the function in the bracket is universal for gapless fermion superfluids in equal mass systems. This function appears in the Meissner mass squared for the 8th gluon in two flavor gapless color superconductor[20, 21]. In some anisotropic states in equal mass systems such as the LOFF state[26, 35] and the BP state via p-wave pairing[40], a similar function appears. In the LOFF state $\delta\mu$ is replaced by an angle dependent mismatch δ_θ [26, 35], and in the p-wave BP state the gap Δ is replaced by an anisotropic gap function $\Delta_{\mathbf{n}}$ [40].

In general case with $m_a \neq m_b$, we define a mass ratio $\lambda = m_b/m_a$ and set $\lambda > 1$ without loss of generality. For $\lambda \neq 1$, the results for $p_a < p_b$ and $p_a > p_b$ (or $n_a < n_b$ and $n_a > n_b$) are not symmetric. We will discuss these two cases separately at zero temperature. For the sake of simplicity, we set $Q_a = Q_b = 1$ in the calculations.

A. $p_a < p_b (n_a < n_b)$

In this case, the branch $\epsilon_{\mathbf{p}}^B$ becomes gapless and we have $n_a(\mathbf{p}) = 0, n_b(\mathbf{p}) = 1$ in the region $p_1 < p < p_2$. At zero temperature, the superfluid density in the BP state can be evaluated as

$$\rho_s = m_a \frac{\alpha_s p_1^3 + \beta_s p_2^3}{6\pi^2} \quad (70)$$

with the coefficients α_s and β_s defined as

$$\begin{aligned} \alpha_s &= 1 - \frac{\lambda}{|1 - (\lambda + 1)v_1^2|} - 3(\lambda + 1) \int_{R_1} dp \frac{p^2}{p_1^3} u_p^2, \\ \beta_s &= \lambda - \frac{\lambda}{|1 - (\lambda + 1)v_2^2|} + 3(\lambda + 1) \int_{R_2} dp \frac{p^2}{p_2^3} v_p^2 \end{aligned} \quad (71)$$

where v_1^2 and v_2^2 are the values of v_p^2 at $p = p_1$ and $p = p_2$, and the integral regions R_1 and R_2 are $0 < p < p_1$ and

$p_2 < p < \infty$ respectively. The Meissner mass squared in the BP state can be evaluated as

$$m_A^2 = \frac{e^2}{m_b} \frac{\alpha_m p_1^3 + \beta_m p_2^3 - \gamma_m p_F^3}{6\pi^2} \quad (72)$$

where the coefficients α_m, β_m and γ_m are defined as

$$\begin{aligned} \alpha_m &= \lambda - \frac{[1 + (\lambda - 1)v_1^2]^2}{|1 - (\lambda + 1)v_1^2|} - 3(\lambda + 1) \int_{R_1} dp \frac{p^2}{p_1^3} u_p^2, \\ \beta_m &= 1 - \frac{[1 + (\lambda - 1)v_2^2]^2}{|1 - (\lambda + 1)v_2^2|} + 3(\lambda + 1) \int_{R_2} dp \frac{p^2}{p_2^3} v_p^2, \\ \gamma_m &= \frac{(\lambda - 1)^2}{\lambda + 1} \int_{R_1 + R_2} dp \frac{p^2}{p_F^3} \frac{\Delta^2 \xi_p}{[(\xi_p - \mu)^2 + \Delta^2]^{3/2}}. \end{aligned} \quad (73)$$

From $p_a < p_b$, we have $\lambda\mu_b > \mu_a$, and the chemical potentials in the BP state satisfy

$$\frac{\Delta_0}{2} < \frac{\lambda\mu_b - \mu_a}{2\sqrt{\lambda}} < \Delta_0. \quad (74)$$

Without loss of generality, we can keep μ_b fixed. After a simple algebra we find that

$$\lambda\mu_b - 2\sqrt{\lambda}\Delta_0 < \mu_a < \lambda\mu_b - \sqrt{\lambda}\Delta_0. \quad (75)$$

The lower bound corresponds to $\Delta = \Delta_0$ where $\alpha = 0$, and the upper bound corresponds to $\Delta = 0$ where $\alpha = \alpha_c$. Then we can calculate the superfluid density and the Meissner mass squared as functions of Δ/Δ_0 in the BP range $0 < \Delta/\Delta_0 < 1$. In Fig.1, we show the superfluid density and Meissner mass squared for different values of mass ratio λ . We found that the superfluid density is always negative at any mass ratio, but the Meissner mass squared is positive in the region $0 < \Delta/\Delta_0 < \nu$ with $\nu < 1$. When the mass ratio becomes very large, such as $\lambda = 100$, ν is close to 1. Even though there exists a big room where the Meissner mass squared is positive, both the superfluid density and the Meissner mass squared tend to negative infinity near $\Delta/\Delta_0 = 1$. Such a divergence at the BP-BCS transition point, which comes from the divergent density of state of the gapless excitations, can not be avoided[17, 34].

B. $p_a > p_b (n_a > n_b)$

In this case, the branch $\epsilon_{\mathbf{p}}^A$ becomes gapless and we have $n_a(\mathbf{p}) = 1, n_b(\mathbf{p}) = 0$ in the region $p_1 < p < p_2$. At zero temperature, the superfluid density in the BP state takes the same form (70) but with different coefficients

$$\begin{aligned} \alpha_s &= \lambda - \frac{\lambda}{|\lambda - (\lambda + 1)v_1^2|} - 3(\lambda + 1) \int_{R_1} dp \frac{p^2}{p_1^3} u_p^2, \\ \beta_s &= 1 - \frac{\lambda}{|\lambda - (\lambda + 1)v_2^2|} + 3(\lambda + 1) \int_{R_2} dp \frac{p^2}{p_2^3} v_p^2 \end{aligned} \quad (76)$$

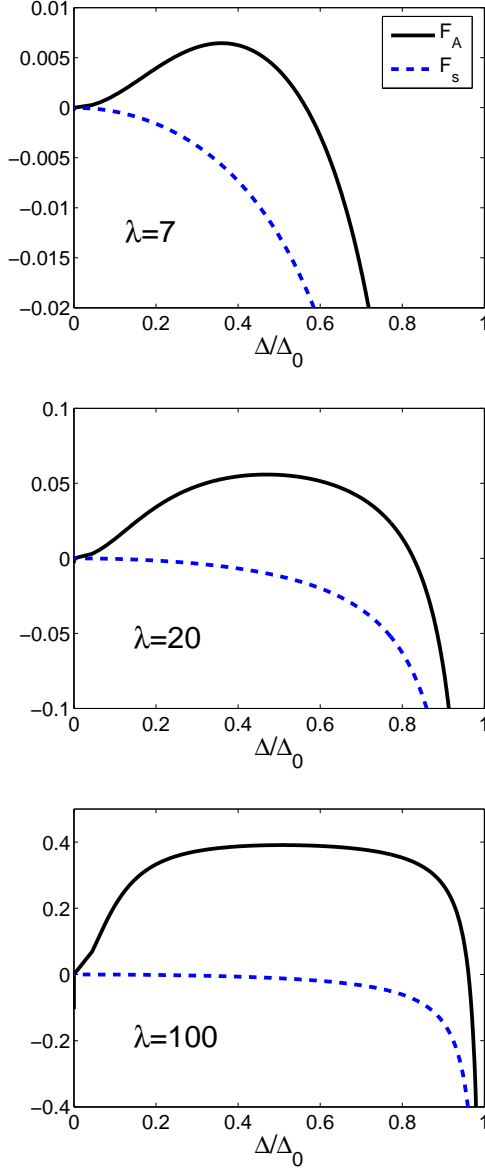


FIG. 1: The scaled Meissner mass squared $F_A = m_A^2/m_0^2$ (solid lines) and superfluid density $F_s = \rho_s/\rho_0$ (dashed lines) as functions of Δ/Δ_0 for different values of mass ratio λ in the case $p_a < p_b$. The normalization constants m_0^2 and ρ_0 are chosen to be $m_0^2 = e^2 m^2 p_b^3 / (m_a^2 m_b)$ and $\rho_0 = m_b p_b^3$. The BCS gap Δ_0 is chosen to be $\Delta_0 = 0.01\mu_b$.

The Meissner mass squared takes also the same form (72), but the coefficients α_m, β_m are modified to

$$\begin{aligned} \alpha_m &= 1 - \frac{[\lambda - (\lambda - 1)v_1^2]^2}{|\lambda - (\lambda + 1)v_1^2|} - 3(\lambda + 1) \int_{R_1} dp \frac{p^2}{p_1^3} u_p^2, \\ \beta_m &= \lambda - \frac{[\lambda - (\lambda - 1)v_2^2]^2}{|\lambda - (\lambda + 1)v_2^2|} + 3(\lambda + 1) \int_{R_2} dp \frac{p^2}{p_2^3} v_p^2, \end{aligned} \quad (77)$$

and γ_m remains unchanged.

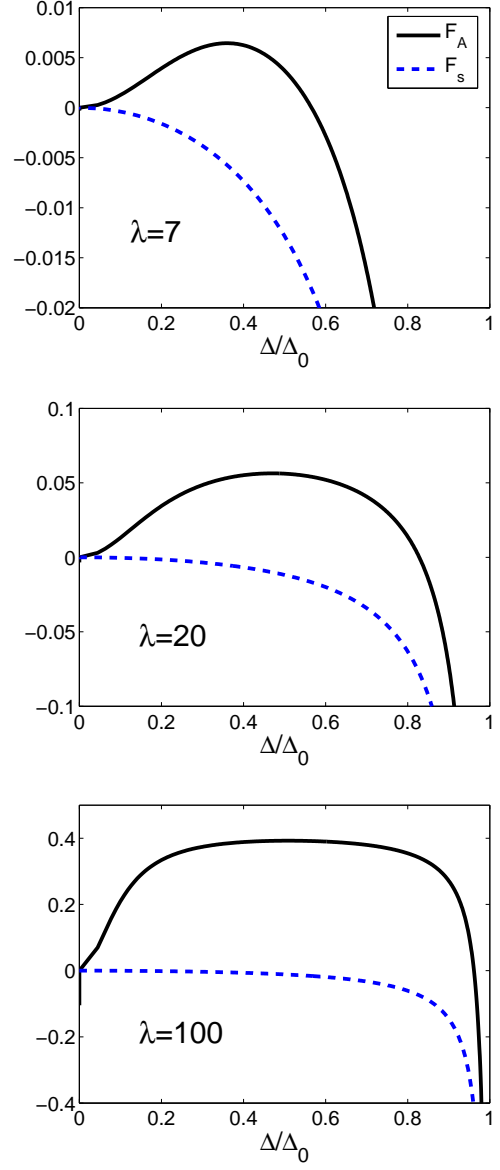


FIG. 2: The scaled Meissner mass squared F_A (solid lines) and superfluid density F_s (dashed lines) as functions of Δ/Δ_0 for different values of mass ratio λ in the case $p_a > p_b$. The BCS gap Δ_0 is chosen to be $\Delta_0 = 0.01\mu_b$.

Similarly, for fixed μ_b , we have

$$\lambda\mu_b + \sqrt{\lambda}\Delta_0 < \mu_a < \lambda\mu_b + 2\sqrt{\lambda}\Delta_0 \quad (78)$$

for the BP state. The superfluid density and the Meissner mass squared are calculated in Fig.2 as functions of Δ/Δ_0 in the range $0 < \Delta/\Delta_0 < 1$. The qualitative behavior is almost the same as in the case $p_a < p_b$.

In summary, we have shown in weak coupling that, the superfluid density of the BP state is always negative, while the Meissner mass squared can be positive in a wide region. The conclusion here is valid for stronger coupling, if there exist two gapless nodes[16].

VI. LOFF STATE

In weak coupling the superfluid density of BP state is always negative, which indicates that the BP state is unstable and some inhomogeneous and anisotropic state is energetically favored. In this section we show that the LOFF state is energetically favored due to the negative superfluid density.

For the sake of simplicity, we consider the simplest pattern of LOFF state, namely the single plane wave ansatz

$$\langle \phi(x) \rangle = \Delta e^{2i\mathbf{q} \cdot \mathbf{x}}, \quad \langle \phi^*(x) \rangle = \Delta e^{-2i\mathbf{q} \cdot \mathbf{x}}, \quad (79)$$

where Δ is a real quantity, and $2\mathbf{q}$ is the so called LOFF momentum which is the total momentum of a Cooper pair. To evaluate the thermodynamic potential of the LOFF state, we often define new fermion fields $\chi_a(x) = e^{i\mathbf{q} \cdot \mathbf{x}} \psi_a(x)$ and $\chi_b(x) = e^{i\mathbf{q} \cdot \mathbf{x}} \psi_b(x)$, and then can directly evaluate the Gaussian path integral in the new basis χ_a, χ_b . Following this way, the thermodynamic potential reads

$$\Omega = \frac{\Delta^2}{g} - T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \text{Tr} \ln \mathcal{G}_q^{-1}(i\omega_n, \mathbf{p}) \quad (80)$$

in terms of the new inverse propagator

$$\mathcal{G}_q^{-1}(i\omega_n, \mathbf{p}) = \begin{pmatrix} i\omega_n - \epsilon_{\mathbf{p}+\mathbf{q}}^a & \Delta \\ \Delta & i\omega_n + \epsilon_{\mathbf{p}-\mathbf{q}}^b \end{pmatrix}. \quad (81)$$

This is just the $(\mathbf{q} + \mathbf{p}, \mathbf{q} - \mathbf{p})$ picture of the LOFF pairing, which means the fermions in the cooper pair move together with a total momentum $2\mathbf{q}$. To see whether the LOFF state is energetically favored, we take the small q expansion

$$\Omega(q) - \Omega(0) = \frac{1}{2} \frac{\partial^2 \Omega}{\partial q^2} \Big|_{q=0} q^2 + O(q^4), \quad (82)$$

where we have chosen a suitable z-direction such that $\mathbf{q} = (0, 0, q)$. Notice that a linear term in q vanishes automatically. One can easily observe the following relation between the momentum susceptibility and the Meissner mass squared:

$$m_A^2 = e^2 \frac{\partial^2 \Omega}{\partial q^2} \Big|_{q=0}. \quad (83)$$

For large mass difference, since the Meissner mass squared is positive in the small Δ region which is just the window for LOFF state, as we have shown in the last section, we may conclude that the LOFF state is not energetically favored. However, we shall argue in the following that this is not the truth.

To give a correct argument we focus on the fact that the superfluid density is always negative for any mass ratio. Notice that we can do any transformation like

$$\chi_a(x) = e^{i\mathbf{q}_a \cdot \mathbf{x}} \psi_a(x), \quad \chi_b(x) = e^{i\mathbf{q}_b \cdot \mathbf{x}} \psi_b(x) \quad (84)$$

to evaluate the effective potential, since the phase factor in the condensate can be eliminated by any \mathbf{q}_a and \mathbf{q}_b satisfying $\mathbf{q}_a + \mathbf{q}_b = 2\mathbf{q}$. For such a general transformation, the thermodynamic potential reads

$$\Omega = \frac{\Delta^2}{g} - T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \text{Tr} \ln \mathcal{G}_{q_a, q_b}^{-1}(i\omega_n, \mathbf{p}) \quad (85)$$

with

$$\mathcal{G}_{q_a, q_b}^{-1}(i\omega_n, \mathbf{p}) = \begin{pmatrix} i\omega_n - \epsilon_{\mathbf{p}+\mathbf{q}_a}^a & \Delta \\ \Delta & i\omega_n + \epsilon_{\mathbf{p}-\mathbf{q}_b}^b \end{pmatrix}. \quad (86)$$

This arbitrariness of phase transformation is directly linked to the fact that the symmetry group of the model Lagrangian is $U(1)_{\varphi_a} \otimes U(1)_{\varphi_b}$. To link the LOFF state and the superfluid density, we introduce a LOFF velocity \mathbf{w} such that

$$\mathbf{q}_a = m_a \mathbf{w}, \quad \mathbf{q}_b = m_b \mathbf{w}. \quad (87)$$

With a suitable choice of coordinates such that $\mathbf{w} = (0, 0, w)$, we can do the similar small w expansion

$$\Omega(w) - \Omega(0) = \frac{1}{2} \frac{\partial^2 \Omega}{\partial w^2} \Big|_{w=0} w^2 + O(w^4). \quad (88)$$

Also, one can easily observe the following relation between the velocity susceptibility and the superfluid density:

$$\rho_s = \frac{\partial^2 \Omega}{\partial w^2} \Big|_{w=0}. \quad (89)$$

This intuitive argument indicates that the energetically favored momentum configuration of LOFF state is $\mathbf{q}_a = m_a \mathbf{w}, \mathbf{q}_b = m_b \mathbf{w}$, which means the single plane wave LOFF ansatz is nothing but the anisotropic state with spontaneously generated superflow \mathbf{v}_s if we identify $\mathbf{w} = \mathbf{v}_s$. In fact, it is quite easy for us to understand this fact. The physical picture of the LOFF state is that the fermions in a cooper pair move together with a nonzero momentum, and hence they should possess a same velocity, not momentum. In fact, we have checked numerically that for general choice of \mathbf{q}_a and \mathbf{q}_b that the quadratic term in the expansion is always negative only when $\mathbf{q}_a = m_a \mathbf{w}, \mathbf{q}_b = m_b \mathbf{w}$.

With the proper configuration $\mathbf{q}_a = m_a \mathbf{w}, \mathbf{q}_b = m_b \mathbf{w}$, we can evaluate the effective potential as

$$\begin{aligned} \Omega(\Delta, w) &= \frac{\Delta^2}{g} - \int \frac{d^3\mathbf{p}}{(2\pi)^3} (E_\Delta - E_S) \\ &\quad - \int \frac{d^3\mathbf{p}}{(2\pi)^3} [H(E_{\mathbf{p}}^A) + H(E_{\mathbf{p}}^B)]. \end{aligned} \quad (90)$$

Here $H(x) = T \ln(1 + e^{-x/T})$, $E_S = \xi_p - \mu_w$, $E_\Delta = \sqrt{E_S^2 + \Delta^2}$ and $E_{\mathbf{p}}^A, E_{\mathbf{p}}^B$ are the energies of the quasiparticles

$$E_{\mathbf{p}}^{A,B} = E_\Delta \pm \left(\frac{p^2}{2m'} + \delta_w + \mathbf{p} \cdot \mathbf{w} \right) \quad (91)$$

with $\mu_w = \mu - (m_a + m_b)w^2/4$ and $\delta_w = \delta\mu + (m_a - m_b)w^2/4$. Notice that a new kinetic energy term $(m_a - m_b)w^2/4$ which vanishes in the equal mass case appears in the asymmetric part of the quasiparticle dispersions. In fact, this momentum configuration is the most convenient one to calculate the LOFF solution, since the anisotropic term $\mathbf{p} \cdot \mathbf{w}$ appears only in the asymmetric part.

Now we give a preliminary discussion on the LOFF state with mass difference. For convenience, we assume the pairing interaction is restricted in the region $p_F - \Lambda < |\mathbf{p}| < p_F + \Lambda$ with $\Lambda \ll p_F$, here Λ serves as a natural ultraviolet cutoff in the theory. In weak coupling we can safely neglect the terms of order $O(w^2)$ and do the following replacement

$$\mathbf{p} \cdot \mathbf{w} \rightarrow p_F w \cos \theta, \quad \frac{p^2}{2m'} \rightarrow \frac{p_F^2}{2m'}, \quad (92)$$

where θ is angle between \mathbf{p} and \mathbf{w} . Up to now, all things become the same as those in the equal mass systems[4, 26], and the conclusions there can be directly applied. If the chemical potentials for the two species are fixed, the corresponding LOFF window is

$$0.707\Delta_0 < \left| \delta\mu + \frac{p_F^2}{2m'} \right| < 0.754\Delta_0, \quad (93)$$

where Δ_0 is the BCS gap, and the LOFF velocity w is approximately given by[4, 26]

$$p_F w \simeq 1.2 \left| \delta\mu + \frac{p_F^2}{2m'} \right|. \quad (94)$$

Defining the mass asymmetry $\epsilon = (m_b - m_a)/(m_b + m_a)$ we have

$$\delta\mu + \frac{p_F^2}{2m'} = \frac{1}{2} [(1 + \epsilon)\mu_b - (1 - \epsilon)\mu_a] \equiv \delta(\epsilon), \quad (95)$$

and then can reexpress the LOFF window as the conventional form in equal mass case

$$0.707\Delta_0 < |\delta(\epsilon)| < 0.754\Delta_0 \quad (96)$$

and $p_F w \simeq 1.2\delta(\epsilon)$. Due to the relation

$$\delta(\epsilon) = \frac{p_b^2 - p_a^2}{2(m_a + m_b)}, \quad (97)$$

the size of the LOFF momentum is

$$|2\mathbf{q}| = (m_a + m_b)w \simeq 1.2(p_b - p_a), \quad (98)$$

which is just we expect. A LOFF state induced by a pure mass difference is of great interest since in some physical systems the chemical potentials are always equal due to chemical equilibrium. Setting $\mu_a = \mu_b \equiv \mu$ we obtain the mass difference window

$$0.707\frac{\Delta_0}{\mu} < |\epsilon| < 0.754\frac{\Delta_0}{\mu}. \quad (99)$$

In weak coupling, $\Delta_0 \ll \mu$, the LOFF state can exist only when the mass asymmetry is very small, which may be realized in electronic systems. If the particle number densities n_a and n_b are fixed, we should compare the free energy $\mathcal{F} = \Omega + \mu_a n_a + \mu_b n_b$. In this case we have the similar expansion $\mathcal{F}(w) = \mathcal{F}(0) + \rho_s w^2/2 + O(w^4)$ which means the LOFF state is more stable than BP state. The LOFF window will be larger, which is similar to the equal mass system[35, 36]. Such a situation may be realized in cold atomic Fermi gas, such as a mixture of ^6Li and ^{40}K atoms.

Finally, we calculate the superfluid density tensor and the Meissner mass squared tensor in the LOFF state. Since the rotational symmetry $O(3)$ is broken down to $O(2)$, the superfluid density and Meissner mass squared become tensors ρ_{ij} and $(m_A^2)_{ij}$. We can decompose them into a transverse part and a longitudinal part

$$\begin{aligned} \rho_{ij} &= \rho_T(\delta_{ij} - \hat{w}_i \hat{w}_j) + \rho_L \hat{w}_i \hat{w}_j, \\ (m_A^2)_{ij} &= m_T^2(\delta_{ij} - \hat{w}_i \hat{w}_j) + m_L^2 \hat{w}_i \hat{w}_j \end{aligned} \quad (100)$$

with $\hat{w} \equiv \mathbf{w}/|\mathbf{w}|$. The transverse and longitudinal superfluid density read

$$\begin{aligned} \rho_T &= m_a n_a + m_b n_b + \frac{3}{4} \int_{-1}^1 d\cos\theta \sin^2\theta F(\cos\theta), \\ \rho_L &= m_a n_a + m_b n_b - \frac{3}{2} \int_{-1}^1 d\cos\theta \cos^2\theta F(\cos\theta) \end{aligned} \quad (101)$$

with the function $F(\cos\theta)$ defined as

$$F(\cos\theta) = \int_0^\infty dp \frac{p^4}{4\pi^2} [f'(E_{\mathbf{p}}^A) + f'(E_{\mathbf{p}}^B)], \quad (102)$$

while the transverse and longitudinal Meissner mass squared read

$$\begin{aligned} m_T^2 &= e^2 \left(\frac{n_a}{m_a} + \frac{n_b}{m_b} \right) + \frac{3e^2}{4} \int_{-1}^1 d\cos\theta \sin^2\theta G(\cos\theta), \\ m_L^2 &= e^2 \left(\frac{n_a}{m_a} + \frac{n_b}{m_b} \right) + \frac{3e^2}{2} \int_{-1}^1 d\cos\theta \cos^2\theta G(\cos\theta) \end{aligned} \quad (103)$$

with the function $G(\cos\theta)$ defined as

$$\begin{aligned} G(\cos\theta) &= \int_0^\infty dp \frac{p^4}{4\pi^2} \frac{4u_p^2 v_p^2}{m'} \frac{f(E_{\mathbf{p}}^A) + f(E_{\mathbf{p}}^B) - 1}{\epsilon_\Delta} \\ &+ \int_0^\infty dp \frac{p^4}{4\pi^2} \left(\frac{u_p^2}{m_a} + \frac{v_p^2}{m_b} \right)^2 f'(E_{\mathbf{p}}^A) \\ &+ \int_0^\infty dp \frac{p^4}{4\pi^2} \left(\frac{v_p^2}{m_a} + \frac{u_p^2}{m_b} \right)^2 f'(E_{\mathbf{p}}^B). \end{aligned} \quad (104)$$

In equal mass systems, we have $m_T^2 \propto \rho_T$ and can prove they are both zero[26], which means that there are

no transverse Meissner effect and superfluid density. The reason is that the formula of the transverse Meissner mass squared is just the gap equation for the LOFF momentum, see[4, 26]. However, for unequal mass systems, the gap equation seems the same as equal mass systems, but the formula of Meissner mass squared becomes quite different, and there are both transverse and longitudinal Meissner effects.

VII. EXTENSION TO FINITE RANGE INTERACTION

The formulae for the superfluid density and Meissner mass squared we derived are based on the point interaction model (1). In this section, we show that the formula can be directly applied to finite range interaction systems, if we replace the constant gap Δ by a momentum-dependent gap function $\Delta(\mathbf{p})$.

With a finite range interaction, the Lagrangian can be written as

$$\begin{aligned} \mathcal{L} = & \int d^3\mathbf{x} \sum_{i=a,b} \psi_i^*(\mathbf{x}, \tau) \left(-\partial_\tau + \frac{\nabla^2}{2m_i} + \mu_i \right) \psi_i(\mathbf{x}, \tau) \\ & + \int d^3\mathbf{x} d^3\mathbf{y} \psi_a^*(\mathbf{x}) \psi_b^*(\mathbf{y}) V(\mathbf{x}, \mathbf{y}) \psi_b(\mathbf{y}) \psi_a(\mathbf{x}), \end{aligned} \quad (105)$$

where we have assumed that the interaction is static. For convenience, we define the condensates

$$\begin{aligned} \Phi(\mathbf{x}, \mathbf{y}) &= \langle \psi_b(\mathbf{y}) \psi_a(\mathbf{x}) \rangle, \\ \Phi^*(\mathbf{x}, \mathbf{y}) &= \langle \psi_a^*(\mathbf{x}) \psi_b^*(\mathbf{y}) \rangle, \end{aligned} \quad (106)$$

and the gap functions

$$\begin{aligned} \Delta(\mathbf{x}, \mathbf{y}) &= V(\mathbf{x}, \mathbf{y}) \langle \psi_b(\mathbf{y}) \psi_a(\mathbf{x}) \rangle, \\ \Delta^*(\mathbf{x}, \mathbf{y}) &= V(\mathbf{x}, \mathbf{y}) \langle \psi_a^*(\mathbf{x}) \psi_b^*(\mathbf{y}) \rangle. \end{aligned} \quad (107)$$

If the system is translational invariant with $V(\mathbf{x}, \mathbf{y}) = V(\mathbf{x} - \mathbf{y})$, Φ, Δ and their complex conjugates depend only on $\mathbf{x} - \mathbf{y}$. In mean field approximation, the thermodynamic potential can be evaluated as

$$\begin{aligned} \Omega = & -T \sum_n \int \frac{d^3\mathbf{p}}{(2\pi)^3} \text{Tr} \ln \mathcal{G}^{-1}(i\omega_n, \mathbf{p}) \\ & + \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int \frac{d^3\mathbf{q}}{(2\pi)^3} \Phi(\mathbf{p}) \Phi^*(\mathbf{q}) V(\mathbf{p} - \mathbf{q}) \end{aligned} \quad (108)$$

in terms of the inverse fermion propagator

$$\mathcal{G}^{-1}(i\omega_n, \mathbf{p}) = \begin{pmatrix} i\omega_n - \epsilon_{\mathbf{p}}^a & \Delta(\mathbf{p}) \\ \Delta^*(\mathbf{p}) & i\omega_n + \epsilon_{\mathbf{p}}^b \end{pmatrix}, \quad (109)$$

where $V(\mathbf{p}), \Phi(\mathbf{p})$ and $\Delta(\mathbf{p})$ are Fourier transformation of $V(\mathbf{x} - \mathbf{y}), \Phi(\mathbf{x} - \mathbf{y})$ and $\Delta(\mathbf{x} - \mathbf{y})$. Since the derivation of the superfluid density and Meissner mass squared depend only on the fermion propagator \mathcal{G} , we conclude that the formulae for superfluid density and Meissner mass

squared derived in Sections V and VI are still valid in the finite range interaction model, if we replace the constant gap Δ by the momentum-dependent gap function $\Delta(\mathbf{p})$.

The BP state with zero range interaction suffers negative superfluid density, and is hence ruled out. It was proposed that the BP state may be stable in a finite range interaction model with large mass ratio[12], since it is the global minimum of the thermodynamic potential with fixed chemical potentials. For a complete study, we check now the superfluid density.

For a spherically symmetric potential $V(r)$, the gap function depends only on $|\mathbf{p}|$ and satisfies the integral equation

$$\Delta(q) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} V(|\mathbf{q} - \mathbf{p}|) \frac{\Theta(\epsilon_{\mathbf{p}}^A) - \Theta(-\epsilon_{\mathbf{p}}^B)}{2\sqrt{(\xi_p - \mu)^2 + \Delta^2(p)}} \Delta(p). \quad (110)$$

For a given potential V and Fermi surface mismatch, we can solve the equation and then determine the ground state with the lowest thermodynamic potential. Once the BP solution is obtained, we can calculate the superfluid density and Meissner mass squared.

For simplicity, let us concentrate on the superfluid density in the case with $p_b > p_a$. The superfluid density is still in the form (70) but the coefficients become

$$\begin{aligned} \alpha_s &= 1 - \frac{\lambda}{|g(p_1) - (\lambda + 1)v_1^2|} - 3(\lambda + 1) \int_0^{p_1} dp \frac{p^2}{p_1^3} u_p^2, \\ \beta_s &= \lambda - \frac{\lambda}{|g(p_2) - (\lambda + 1)v_2^2|} + 3(\lambda + 1) \int_{p_2}^\infty dp \frac{p^2}{p_2^3} v_p^2, \end{aligned} \quad (111)$$

where the function $g(p)$ is defined as

$$g(p) = 1 + \frac{m_b}{p} \frac{\Delta(p)\Delta'(p)}{\sqrt{(\xi_p - \mu)^2 + \Delta^2(p)}}, \quad (112)$$

with $\Delta'(p) = d\Delta(p)/dp$. For the point interaction model, we have $\Delta'(p) = 0$ and hence $g(p) = 1$ which leads to negative superfluid density, as we discussed above.

Generally, the gap function peaks at $p = p_0$ and drops down fast for $p > p_0$. In this case, we have $g(p) = 1$ and $v_p^2 = 0$ for $p \geq p_2$ [12], and the sign of the superfluid density depends only on α_s . The condition to produce a BP state with positive superfluid density is then

$$|g(p_1) - (\lambda + 1)v_1^2| > \frac{\lambda}{1 - 3(\lambda + 1)p_1^{-3} \int_0^{p_1} dp p^2 u_p^2}. \quad (113)$$

If the slope of the gap function at $p = p_1$ is very large, the condition can be easily satisfied.

For the interaction with a momentum cutoff p_Λ [12], the momentum structure of the gap function is $\Delta(p) = \Delta$ for $p < p_\Lambda$ and $\Delta(p) = 0$ for $p > p_\Lambda$, and we have $g(p) = 1$ for all p except at $p = p_\Lambda$. Since in general case the positions of the zero nodes are not exactly located at the cutoff, $p_{1,2} \neq p_\Lambda$, the situation in this model is just the same as in the point interaction model.

VIII. CONCLUDING REMARKS

We have derived the superfluid density and the Meissner mass squared for the fermion cooper pairing with unequal masses via a standard field theory approach. For equal mass systems, the two variables are indeed proportional to each other, while for unequal mass systems, this relation breaks down. In the breached pairing states with zero range interaction, the superfluid density is always negative, but the Meissner mass squared is positive in a wide region. As a consequence, the momentum configuration of the LOFF pairing should be correctly established. We propose a proper momentum configuration for LOFF pairing with unequal masses and show that the single plane wave LOFF configuration in unequal mass system is physically equivalent to an anisotropic state with spontaneously generated superflow. These conclusions are valid only in weak coupling. Whether they are valid for stronger coupling, especially in the BCS-BEC crossover, should be examined.

There are some problems related to the arbitrariness in the phase transformation induced by the $U(1)_{\varphi_a} \otimes U(1)_{\varphi_b}$ symmetry. To investigate the Goldstone mode or the phase fluctuation in the superfluid state, we often neglect the fluctuation of the amplitude of the order parameter and write $\phi(x) = \Delta e^{2i\theta(x)}$. Using the standard phase transformation $\psi_i(x) = \tilde{\psi}_i(x) e^{i\theta(x)}$ we can obtain the effective action for the Goldstone boson. However, generally we can transform the fermion fields as $\psi_i(x) = \tilde{\psi}_i(x) e^{i\nu_i\theta(x)}$ with ν_a and ν_b arbitrary constants satisfying the constraint $\nu_a + \nu_b = 2$. The low energy effective action for the phase field $\theta(x)$ generally reads

$$S_{eff}[\theta] = -\frac{1}{2} \sum_{\mathbf{q}} (\mathcal{D}q_0^2 - \mathcal{P}\mathbf{q}^2) |\theta(q_0, \mathbf{q})|^2. \quad (114)$$

Only when all fermionic excitations are gapped and at weak coupling limit, we find \mathcal{D} and \mathcal{P} are independent of ν_a and ν_b . Hence the result of Goldstone boson velocity in [41] is safe. This problem may imply that we can not safely neglect the fluctuation of the amplitude of the order parameter in strong coupling or in the gapless phases.

Another problem is the stability condition related to the phase fluctuation. The superfluid density ρ_s is often regarded as a quantity to judge the stability of BP state[13]. For simplicity we focus on the equal mass case. When ρ_s is negative, it directly means that the LOFF state has lower energy than the BP state. However, this is true only for the standard LOFF state with $\mathbf{q}_a = \mathbf{q}_b$. For general case with $\mathbf{q}_a = \nu_a \mathbf{q}$ and $\mathbf{q}_b = \nu_b \mathbf{q}$, we should check the sign of $\kappa_q = \partial^2 \Omega / \partial q^2|_{q=0}$ for all possible ν_a and ν_b . While ρ_s is positive in strong coupling BEC region which means BP state is stable against the standard LOFF state with $\mathbf{q}_a = \mathbf{q}_b$ [13], there is no direct observation that κ_q is positive for any ν_a and ν_b , such as $\nu_a = 2, \nu_b = 0$. If κ_q becomes negative for $\nu_a \neq \nu_b$, a non-standard LOFF state with $\mathbf{q}_a \neq \mathbf{q}_b$ is energetically favored in strong coupling.

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